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Algorithms

**Bitonic Travelling Salesman**

***Travelling Salesman Problem*** Finds shortest possible route that visits set of cities exactly once and returns to starting city; ***Bitonic Travelling Salesman Problem*** Variation of TSP where salesman must visit cities in bitonic manner, first travelling along monotonic path and then reversing to return to starting city.

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**Evaluation**

***Cost model*** estimates time taken for instructions to be executed. Very general cost model counts number of reductions made.

***Evaluation Order* Applicative Order (strict setting)** leftmost innermost reducible expression (evaluates arguments before function) **Normal Order (lazy setting)** leftmost outermost reducible expression (evaluates function before its arguments). If they terminate both produce values in normal form.

***If normal form for expression exists, normal order will always reduce to that normal form, but applicative order may not find that form as it may not terminate.***

***Counting Carefully***

***Non-primitive function:*** f a1 a2 … an = e => T(f) a1 a2 … an = T(e) + 1; ***Primitive function:*** f x1 … xn = 0 => T(f) x1 … xn = 0; ***Variable:*** x => T(x) = 0; ***Application:*** f e1 … en => T(f e1 … en) = T(f) + T(e1) + … + T(en); ***Conditional:*** T(if p then e1 else e2) = T(p) + if p then T(e1) else T(e2).

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**Asymptotics**

***Logarithmico-exponential function (L-function)*** Real, positive, monotonic, one-valued function on real variable defined for all values greater than some definite value by finite combination of algebraic symbols, exponentials, logarithms, operating on real constants and variable.

***[THEOREM]*** ***Any L-function f is ultimately continuous, of constant sign, monotonic, and as n 🡪 inf, the value f(n) tends to one of 0, inf, or some other definite limit.***

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***Trichotomy*** Theorem mentioned above says one of fg, fg, fg must hold forming a **trichotomy**.

***Converse*** fg ⬄ fg; ***Transitive*** fg of gh => fh; fg gh => fh.

1 log n root(n) n n log n n2 n3 n! nn.

***Bachmann-Landau Notation***

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**Amortized Analysis**

***Amortized analysis*** Gives cost of operation in context of sequence of previous operations on data structure.

***Deque*** Double ended queue (symmetric list) is a queue where elements can be added both at front/back efficiently and is in this sense double ended. Deque contains xs and sy, which together form list with all elements in xs followed by reversed elements in sy.



***Du-Bois Reymond Notation***

***Amortization***

***Three things to define*** **Cost function** Cop\_i­­(xs­i) for each operation opi on data xsi; **Amortized cost function**Aop\_i­(xsi) for each operation opi on data xsi; **Size function** S(xs) that calculates sizes of data xs.

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**Abstract Datatypes**

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**Random Access Lists**

***Peano Numbers*** Simplistic way of counting natural numbers, number is either 0 or succ of some other number.

***Binary Numbers*** List of digits with LSB first representation, e.g., [O, O, I, I] = 12.

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***Binary Tree Lookup*** Balanced binary trees allow efficient access to their elements.

***Random Access Lists*** Efficient data structures that combine benefits of lists/binary numbers, allowing for quick random access/modification operations.



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**Divide and Conquer**

***Three parts*** Divide a problem into subproblems, divide and conquer subproblems into sub solutions, conquer sub solutions into solution.

***Merge Sort***

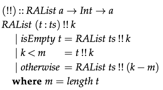
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***Quick Sort***

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**Searching**

***Equality***

***Ordering***

***Search Trees***

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**Dynamic Programming**

***Used to efficiently calculate exact solutions to certain recursive problems (‘trade space for speed’).***

***Memoization*** Storing result of function call so it can be used again later.

***Strategy*** Write inefficient recursive algorithm that solves problem, improve efficiency by storing intermediate shared results.

***Fibonacci example***

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***Array – comes equipped with operation to look up values in constant time, Table, Tabulate***



***Array function*** Builds array from list containing indices and their values, a!i returns value at index i and fails if i out of bounds; ***Table function*** Constructs table in array; ***Tabulate function*** Results of applying f to all values between x/y. Implemented as array so constant time access.

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**Edit-Distance**

***Levenshtein Distance*** between two strings: no of insertions, deletions, updates taken to turn one into other.

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**Useful Haskell Functions**

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**Red-Black Trees**

***They are another means of creating balanced trees. They do not need to store height of current tree (unlike AVL/Binary Search Tree) instead stores colour of node: red/black.***

***Two invariants:*** Every red node must have black parent node; Every path from root to leaf must have same number of black nodes.

***Ensures tree is at most imbalanced by factor of at most two in one of its branches.***

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**Randomized Algorithms**

***Randomized algorithm*** Produce results quickly and with high probability (mostly correct) using random values. **Monte Carlo** Predictable running time/unpredictable correct result; **Las Vegas** Opposite.

***Leibniz’s Law/Identity of indiscernibles*** Functions always map same inputs to same outputs (x=y => fx=fy).

***No function can return truly random result due to Leibniz’s law but can exhibit pseudo-random behaviour (***Depends on some input that varies either explicitly/implicitly***).***

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**Randomized Binary Search Trees**

***Behaves like ordinary binary search tree most of the time, but with some probability will insert a value at its root. Underlying data type is ordinary binary tree.***

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**Master Theorem**

***T(n) = aT(n/b) + f(n) has solutions as follows, where E = log a/ log b is the critical exponent: (1) If nE+ε = O(f(n)) for some ε > 0 then T(n) = Θ(f(n)). (2) If f(n) = Θ(nE) then T(n) = Θ(f(n)logn). (3) If f(n) = O(nE-ε) for some ε > 0 then T(n) = Θ(nE).***

***Examples: Binary Search, MergeSort, Strassen’s Algorithm***

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**Treaps**

***Treap*** Combination of tree and heap, all values to the left are smaller and values to the right are larger, equally parent node has higher priority in heap than children.

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***Randomized Treaps***

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**Array Resizing (Mutable Data Structures Continuation)**

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**Mutable Data Structures Continuation**

***Hashing***

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**Mutable Data Structures**

***Mutable References***

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***Checklist***